

SU(5)-Compatible Yukawaon Model

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Abstract

It is investigated what constraints are imposed on the so-called yukawaon model when we regard quarks and leptons as $(\bar{\mathbf{5}}_i + \mathbf{10}_i + \mathbf{1}_i)$ of SU(5) ($i = 1, 2, 3$: index of U(3) family symmetry). In past yukawaon models, the effective Yukawa coupling constants Y_f^{eff} are given by vacuum expectation values of fields Y_f (“yukawaons”) as $(Y_f^{eff})_{ij} = (y_f/\Lambda)\langle(Y_f)_{ij}\rangle$. In order to build a model without a cutoff scale Λ , vector-like fields $(\bar{\mathbf{5}}^i + \mathbf{10}^i + \mathbf{5}_i + \mathbf{10}_i)$ are introduced in addition to the conventional quarks and leptons. The U(3) family symmetry is broken at $\langle Y_f \rangle \sim 10^{13}$ GeV.

1. Introduction

In the standard model of the quarks and leptons, the mass spectra and mixings are due to the Yukawa coupling constants. Then, one may investigate relations among those fundamental constants by assuming a family symmetry (e.g. a U(1) symmetry, a discrete symmetry, and so on). In contrast to such conventional models, there is another idea: the mass spectra and mixings are due to vacuum expectation values (VEVs) of new scalars. As one of such models, the so-called “yukawaon” model [1] is known.

In the yukawaon model, which is a kind of “flavon” model [2], all effective Yukawa coupling constants Y_f^{eff} ($f = u, d, e, \dots$) are given by VEVs of “yukawaons” Y_f as

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda} \langle (Y_f)_{ij} \rangle, \quad (1.1)$$

where the indices i, j denote $i, j = 1, 2, 3$ in a family symmetry U(3). For example, would-be Yukawa interactions are given by the following superpotential [3]:

$$W_Y = \frac{y_e}{\Lambda} \ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda} \ell_i Y_\nu^{ij} \nu_j^c H_u + \lambda_R \nu_i^c Y_R^{ij} \nu_j^c + \frac{y_u}{\Lambda} u_i^c Y_u^{ij} q_j H_u + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q_j H_d, \quad (1.2)$$

where ℓ and q are SU(2)_L doublets $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$.

Note that the yukawaons Y_f are singlets under the conventional gauge symmetries SU(3)_c × SU(2)_L × U(1)_Y, and they have only family indices. This suggests that the yukawaon model may be compatible with an SU(5) grand unification (GUT) model [4]. For example, we may consider:

$$W_Y = \frac{y_u}{\Lambda} \mathbf{10}_i Y_{(10,10)}^{ij} \mathbf{10}_j \mathbf{5}_H + \frac{y_d}{\Lambda} \bar{\mathbf{5}}_i Y_{(5,10)}^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H + \frac{y_u}{\Lambda} \bar{\mathbf{5}}_i Y_{(5,1)}^{ij} \mathbf{1}_j \mathbf{5}_H + \lambda_R \mathbf{1}_i Y_{(1,1)}^{ij} \mathbf{1}_j, \quad (1.3)$$

where $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$ are quark and lepton fields and $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ correspond to the conventional two Higgs doublets H_u and H_d , respectively. However, in the would-be Yukawa interactions (1.3), the

charged lepton yukawaon Y_e is identical with the down-quark yukawaon Y_d , i.e. $Y_e = Y_d = Y_{(5,10)}$, although Y_u and Y_ν are reasonably given by $Y_u = Y_{(10,10)}$ and $Y_\nu = Y_{(5,1)}$. In the yukawaon model, it is essential that $\langle Y_e \rangle$ has a different family structure from $\langle Y_d \rangle$. We must build a model where yukawaon Y_e is an independent field from Y_d . For this problem, we will introduce matter fields $f' = (\bar{\mathbf{5}}'_i + \mathbf{5}'^i)$ in addition to the quarks and leptons $f_i = (\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1})_i$.

The purpose of the present paper is to investigate what constraints are imposed on the so-called yukawaon model when we regard quarks and leptons as $(\bar{\mathbf{5}}_i + \mathbf{10}_i + \mathbf{1}_i)$ of SU(5). In this paper, we do not try to unify a family symmetry into the SU(5) gauge symmetry. We do not intend to resolve current problems in the minimum SU(5) GUT by considering the yukawaon model. Since yukawaons are SU(5) singlets, the existence of the yukawaons do not affect an SU(5) GUT model, so that we can inherit the successful results in the SU(5) GUT and we also inherit the current problems in the minimum SU(5) GUT scenario. For example, it is well known that it is difficult to build an SU(5) GUT model in which R charges are conserved. However, in the yukawaon model, R charges have to be conserved in order to distinguish yukawaons Y_f from the each other. In the present model, too, we assume R charge conservation, while we do not touch whether this is compatible with SU(5) GUT scenario or not. (Of course, R charge conservation is broken below $\mu = \Lambda_{fam}$, where the family symmetry U(3) is broken at $\mu \sim \Lambda_{fam}$.) We optimistically consider that the problem will be resolved in a scenario with a higher GUT group and/or with an extra-dimension. In the present paper, we pay attention only to yukawaon sector.

So far, as seen in Eq.(1.2), the yukawaon model has been based on an effective theory with a cutoff Λ . Another purpose of the present SU(5) compatible model is to make the meaning of the cutoff scale Λ clear by considering a model without such a cutoff scale. In order to build a model without a cutoff scale Λ , we will introduce matter fields $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$ in addition to the fields f_i and f' . (Note that f'' do not mix with the quarks and leptons f_i because quarks and leptons $f_i = (\bar{\mathbf{5}}(\mathbf{5}') + \mathbf{10} + \mathbf{1})_i$ are $\mathbf{3}$ of U(3), while $(\bar{\mathbf{5}}'' + \mathbf{10}'')^i$ are $\mathbf{3}^*$ of U(3).)

As we discuss later, the existence of the new matter fields f' and f'' will bring some merits and demerits in the yukawaon model: (i) Y_ν in Eq.(1.2) will bring naturally substituted with Y_e in the present model, although the replacement $Y_\nu \rightarrow Y_e$ was an ad hoc ansatz in the previous yukawaon model [3]; (ii) In the past yukawaon model, in order to explain the observed fact $m_t \sim \langle H_u \rangle$, we must consider $\langle Y_u \rangle / \Lambda \sim 1$ (we consider $\tan \beta \sim 10$). This is somewhat controversial within the framework of such an effective theory with Λ . In contrast to the old model, in the present model, the factor $\langle Y_u \rangle / \Lambda$ in the previous model corresponds to $\langle Y_u \rangle / M_{10}$ (M_{10} is a mass parameter in the superpotential and it is given by $M_{10} \bar{\mathbf{10}}''_i \mathbf{10}''^i$), and we can reasonably take $M_{10} \sim \langle Y_u \rangle$. (iii) It is difficult to lower a scale Λ_{fam} of the U(3) family symmetry breaking in the present SU(5) compatible model, in order that the gauge coupling constants of $SU(3)_c \times SU(2)_L \times U(1)_Y$ do not blow up under the additional particles $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$.

In the next section, we investigate possible would-be Yukawa interactions without a cutoff parameter Λ and an answer to the above question (ii) will be given. In Sec.3, we investigate superpotential for the yukawaon sector. Although the VEV relations are substantially inherited

from previous those, the superpotential forms are considerably changed because we do not consider terms which include a cutoff scale Λ . In Sec.4, we discuss a possible scale of Λ_{fam} at which U(3) family symmetry is broken. We will conclude $\Lambda_{fam} \sim 10^{13}$ GeV. Sec.5 is devoted to concluding remarks.

2. Would-be Yukawa interactions

First, let us discuss the Y_e - Y_d splitting. We introduce vector-like $\mathbf{5}'^i$ and $\bar{\mathbf{5}}'_i$ fields in addition to the fields given in Eq.(1.3). For convenience, we denote one $\mathbf{5}$ and two $\bar{\mathbf{5}}$ as

$$\bar{\mathbf{5}}_i = (D_i^c, \ell_i), \quad \bar{\mathbf{5}}'_i = (d_i^c, L_i), \quad \mathbf{5}'^i = (\bar{D}^{ci}, \bar{L}^i), \quad (2.1)$$

where d^c , D^c and \bar{D}^c are SU(2)_L singlet down-quarks with electric charges +1/3, +1/3 and -1/3, respectively, and ℓ , L and \bar{L} are SU(2)_L lepton doublets. In order to realize that the fields (\bar{D}^c, D^c) , and (\bar{L}^c, L^c) become massive and decouple from the present model, we assume the following interactions

$$\lambda_D \bar{\mathbf{5}}_i^A (\Sigma_3)_A^B \mathbf{5}'^i_B + \lambda_L (\bar{\mathbf{5}}'_i)^A (\Sigma_2)_A^B \mathbf{5}'^i_B, \quad (2.2)$$

where indices $A, B = 1, 2, \dots, 5$ are those of SU(5), and SU(5) $\mathbf{24} + \mathbf{1}$ fields Σ_2 and Σ_3 take VEV forms

$$\begin{aligned} \langle \Sigma_2 \rangle &= v_2 \text{diag}(0, 0, 0, 1, 1), \\ \langle \Sigma_3 \rangle &= v_3 \text{diag}(1, 1, 1, 0, 0). \end{aligned} \quad (2.3)$$

Therefore, Eq.(2.2) leads to mass terms

$$\lambda_D v_3 \bar{D}^{ci} D_i^c + \lambda_L v_2 \bar{L}^i L_i. \quad (2.4)$$

As we discuss later, we will assign different R charges to Σ_2 and Σ_3 (and also to $\mathbf{5}$ and $\mathbf{5}'$). If we once admit the existence of Σ_3 with VEV given in Eq.(2.3) phenomenologically, we can use Σ_3 for the triplet-doublet splitting of the Higgs fields $\bar{\mathbf{5}}_H$ and $\mathbf{5}_H$ by considering an interaction $\bar{\mathbf{5}}_H \Sigma_3 \mathbf{5}_H$. For the triplet-doublet splitting in Higgs fields, already a reasonable mechanism has been proposed based on SO(10) GUT scenario [5]. Since the purpose of the present paper is not to investigate the GUT scenario, we ad hoc assume the VEV forms (2.3) in this paper. The reason of the forms (2.3) may be understood by a scenario based on a higher gauge GUT group and/or on extra-dimensions.

Next, we discuss a seesaw-type mass matrix mechanism in order to realize the effective interactions given by Eq.(1.2). We assume further vector-like matter fields $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$. The up-quark, charged lepton, and down-quark mass matrices, M_u , M_e and M_d , are generated by the following superpotential:

$$W_u = y_u \mathbf{10}_i Y_u^{ij} \bar{\mathbf{10}}_j'' + M_{10} \bar{\mathbf{10}}_i'' \mathbf{10}''^i + y_{10} \mathbf{10}''^i \mathbf{10}_i \mathbf{5}_H, \quad (2.5)$$

$$W_{e,d} = y_e \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{5}_j'' + y_d \bar{\mathbf{5}}'_i Y_d^{ij} \mathbf{5}_j'' + M_5 \mathbf{5}_i'' \bar{\mathbf{5}}''^i + y_5 \bar{\mathbf{5}}''^i \mathbf{10}_i \bar{\mathbf{5}}_H, \quad (2.6)$$

which lead to effective Yukawa interactions

$$W_u^{eff} = \frac{y_u y_{10}}{\bar{M}_{10}^{(ij)}} \mathbf{10}_i Y_u^{ij} \mathbf{10}_j \mathbf{5}_H, \quad (2.7)$$

$$W_{e,d}^{eff} = \frac{y_e y_5}{\bar{M}_{5e}^{(ij)}} \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H + \frac{y_d y_5}{\bar{M}_{5d}^{(ij)}} \bar{\mathbf{5}}'_i Y_d^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H, \quad (2.8)$$

respectively. Note that the cutoff parameters have flavor-dependence differently from the conventional seesaw model, because we are interested in cases of $\langle Y_u^{33} \rangle / M_{10} \sim 1$ and $\langle Y_{e,d}^{33} \rangle / M_5 \sim 10^{-1}$.

Here, the factors $\bar{M}_{10}^{(ij)}$, $\bar{M}_{5e}^{(ij)}$ and $\bar{M}_{5d}^{(ij)}$ are given by

$$\bar{M}_{10}^{(ii)} \simeq M_{10} [1 + (\langle Y_u^{ii} \rangle / M_{10})^2], \quad (2.9)$$

$$\bar{M}_{5e}^{(ii)} \simeq \sqrt{(M_5)^2 + (\langle Y_e^{ii} \rangle)^2}, \quad \bar{M}_{5d}^{(ii)} \simeq \sqrt{(M_5)^2 + (\langle Y_d^{ii} \rangle)^2}, \quad (2.10)$$

in the bases in which the VEV matrices $\langle Y_u \rangle$, $\langle Y_e \rangle$ and $\langle Y_d \rangle$ are diagonal, respectively. The expressions (2.9) and (2.10) are obtained from diagonalizing the mass matrices for $(\mathbf{10}, \mathbf{10}'', \bar{\mathbf{10}}'')$ and $(\bar{\mathbf{5}} (\bar{\mathbf{5}}'), \mathbf{10}, \bar{\mathbf{5}}'', \mathbf{5}'')$:

$$M_{10}^u = \frac{1}{2} \begin{pmatrix} 0 & y_{10} v_{Hu} & y_u \langle Y_u \rangle \\ y_{10} v_{Hu} & 0 & M_{10} \\ y_u \langle Y_u \rangle & M_{10} & 0 \end{pmatrix}, \quad M_5^{e,d} = \begin{pmatrix} 0 & 0 & 0 & y_{e,d} \langle Y_{e,d} \rangle \\ 0 & 0 & y_5 v_{Hd} & 0 \\ 0 & y_5 v_{Hd} & 0 & M_5 \\ y_{e,d} \langle Y_{e,d} \rangle & 0 & M_5 & 0 \end{pmatrix}, \quad (2.11)$$

respectively, where $v_{Hu} = \langle H_u^0 \rangle = \langle \mathbf{5}_H \rangle$ and $v_{Hd} = \langle H_d^0 \rangle = \langle \bar{\mathbf{5}}_H \rangle$.

For a neutrino Dirac mass term, we also assume

$$W_\nu = y_e \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{5}_j'' + M_5 \mathbf{5}_i'' \bar{\mathbf{5}}''^i + y_1 \bar{\mathbf{5}}''^i \mathbf{1}_i \mathbf{5}_H, \quad (2.12)$$

which leads to the effective interaction

$$W_\nu^{eff} = \frac{y_e y_1}{M_5} \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{1}_j \mathbf{5}_H. \quad (2.13)$$

(In the present model, we do not assume $\mathbf{1}_i'' + \mathbf{1}''^i$.) Note that the neutrino Dirac mass matrix m_D is proportional to $\langle Y_e \rangle$ as well as the charged lepton mass matrix M_e . When we assume that the SU(5) singlet matter field $\mathbf{1}_i$ can have a Majorana mass term

$$W_R = \lambda_R \mathbf{1}_i Y_R^{ij} \mathbf{1}_j, \quad (2.14)$$

we can obtain a seesaw-type neutrino mass matrix

$$M_\nu = \frac{y_e^2 y_1^2}{\lambda_R} \left(\frac{v_{Hu}}{M_5} \right)^2 \langle Y_e \rangle \langle Y_R \rangle^{-1} \langle Y_e \rangle, \quad (2.15)$$

under an approximation $\bar{M}_{5e} \simeq M_5$, so that we can rewritten Eq.(2.15) as

$$M_\nu = \frac{y_1^2 \tan^2 \beta}{\lambda_R y_5^2} M_e \langle Y_R \rangle^{-1} M_e, \quad (2.16)$$

where $\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$. By taking $m_\tau \simeq 1.777$ GeV, $(M_\nu)_{33} \sim m_{\nu 3} \simeq \sqrt{\Delta m_{atm}^2} \simeq 0.049$ eV [6]) and $\tan \beta \simeq 10$, we can estimate the value of $\langle Y_R \rangle$ as

$$\langle Y_R \rangle \simeq \lambda_R (y_5/y_1)^2 \times 6.4 \times 10^{12} \text{ GeV}. \quad (2.17)$$

R charges of these matter fields must satisfy the following relations:

$$R(\mathbf{10}_i) - R(\mathbf{1}_i) = R(\bar{\mathbf{5}}_i'') - R(\mathbf{10}_i'') = R(\bar{\mathbf{10}}''^i) - R(\mathbf{5}''^i) = R(\mathbf{5}_H) - R(\bar{\mathbf{5}}_H) \equiv a, \quad (2.18)$$

$$R(\bar{\mathbf{5}}_i') - R(\bar{\mathbf{5}}_i) = R(\Sigma_3) - R(\Sigma_2) \equiv b, \quad (2.19)$$

$$R(\bar{\mathbf{5}}''^i) + R(\mathbf{5}_i'') = 2, \quad (2.20)$$

$$R(\bar{\mathbf{5}}_i) + R(\mathbf{5}'^i) + R(\Sigma_3) = 2, \quad (2.21)$$

$$R(Y_R) + 2R(\mathbf{1}_i) = 2. \quad (2.22)$$

Since there are still free parameters in these assignments, we do not show an explicit assignment of R charges in this paper.

3. Superpotential for yukawaons

The successful results in the previous yukawaon model with an $O(3)$ family symmetry [3] have been derived on the basis of the following VEV relations:

$$\langle Y_e \rangle = k_e \langle \Phi_e \rangle \langle \Phi_e \rangle, \quad (3.1)$$

$$\langle Y_u \rangle = k_u \langle \Phi_u \rangle \langle \Phi_u \rangle, \quad (3.2)$$

$$\langle \Phi_u \rangle = k_u' \langle \Phi_e \rangle (\langle E \rangle + a_u \langle X \rangle) \langle \Phi_e \rangle, \quad (3.3)$$

$$\langle Y_d \rangle = k_d \langle \Phi_e \rangle (\langle E \rangle + a_u \langle X \rangle) \langle \Phi_e \rangle, \quad (3.4)$$

$$\langle Y_R \rangle = k_R [\langle \Phi_u \rangle \langle P_u \rangle \langle Y_e \rangle + \langle Y_e \rangle \langle P_u \rangle \langle \Phi_u \rangle + \xi_\nu (\langle \Phi_u \rangle \langle Y_e \rangle \langle P_u \rangle + \langle P_u \rangle \langle Y_e \rangle \langle \Phi_u \rangle)], \quad (3.5)$$

where

$$\langle \Phi_e \rangle_e \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \quad (3.6)$$

$$\langle E \rangle_e = v_E \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \langle X \rangle_e = \frac{1}{3} v_X \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \langle P_u \rangle_u = v_{Pu} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.7)$$

Those VEV relations have been derived from superpotential by using SUSY vacuum conditions, so that the relations are dependent on the quantum number assignments of the yukawaons. In other words, the relations are based on completely phenomenological assumptions (quantum number assignments). The bilinear form of Y_e in Eq.(3.1) is required to give a charged lepton mass relation (see Eq.(3.9) later). The bilinear form of Y_u in Eq.(3.2) plays an inevitable role in obtaining a successful neutrino mass matrix via Eq.(3.5).

Note that the VEV matrix forms (3.6) and (3.7) are dependent on the flavor basis. The expression $\langle A \rangle_f$ ($f = e, u$) denotes a form of the VEV matrix $\langle A \rangle$ in a basis in which the

mass matrix M_f is diagonal. In this paper, we do not give a superpotential which leads to the reasonable eigenvalues (3.6), and we will use the observed running mass values at $\mu = m_Z$ of the charged lepton masses. The VEV forms given in Eq.(3.7) are essentially based on phenomenological assumptions as we state later.

Since Y_e was substituted for Y_ν in the previous model [3], the neutrino mass matrix M_ν was given by the form (2.15). Therefore, the observed neutrino mixing originates only in the structure of $\langle Y_R \rangle$. When a reasonable structure of $\langle Y_R \rangle$ is assumed, the model can give excellent agreement with the nearly tribimaximal mixing [7] together with the up-quark mass ratios only by adjusting two parameters [a_u in Eq.(3.3) and ξ_ν in Eq.(3.5)]. Besides, the model can give rough agreement with the Cabibbo-Kobayashi-Maskawa (CKM) mixing parameters and down-quark mass ratios by adjusting remaining two parameters [the magnitude and phase of a_d in Eq.(3.4)].

In the previous model [3], these VEV relations (3.1) – (3.5) have been derived from the SUSY vacuum conditions by assuming an O(3) family symmetry. However, the superpotential terms have (1.2) included a cutoff parameter Λ . As seen in the previous section, in the present model, we want to build a model without such a cutoff parameter Λ . Moreover, in the present paper, we will investigate a model with a U(3) family symmetry instead of the O(3) family symmetry, because, in the yukawaon model, the order of the VEV matrices is important [e.g. absence of $(E + aX)\Phi_e\Phi_e + \Phi_e\Phi_e(E + aX)$ in contrast to the existence of $\Phi_e(E + aX)\Phi_e$].

Recently, Sumino has proposed a charged lepton mass matrix model based on a U(3) gauge family symmetry [8]. In the Sumino model, the charged lepton mass term is generated by a would-be Yukawa interaction

$$H_e = \frac{y_e}{\Lambda^2} \bar{\ell}_L^i \Phi_{i\alpha}^e \Phi_{\alpha j}^{eT} e_R^j H, \quad (3.8)$$

where i and α are indices of U(3) and O(3), respectively, and H is the Higgs scalar in the standard non-SUSY model. (Sumino' model has not been based on a SUSY scenario.) The charged lepton masses m_{e_i} are acquired from a VEV of the scalar Φ^e [9], i.e. the masses m_{e_i} are given by $m_{e_i} = (y_e/\Lambda^2) \langle \Phi_{i\alpha}^e \rangle \langle \Phi_{\alpha i}^{eT} \rangle \langle H^0 \rangle$. Sumino's interest was in the charged lepton mass relation [10]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \quad (3.9)$$

The relation $K = 2/3$ is satisfied with the order of 10^{-5} for the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [6], while it is only valid with the order of 10^{-3} for the running masses, e.g. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. However, in conventional mass matrix models, “mass” means not “pole mass” but “running mass”. Sumino has seriously taken why the mass formula $K = 2/3$ is so remarkably satisfied with the pole masses. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{e_i} \log(\mu/m_{e_i})$ in the radiative correction term [11] due to photon

$$m_{e_i}(\mu) = m_{e_i}^{pole} \left[1 - \frac{\alpha(\mu)}{\pi} \left(1 + \frac{3}{4} \log \frac{\mu}{m_{e_i}(\mu)} \right) \right]. \quad (3.10)$$

Therefore, he has assumed that a family symmetry is local, and that the logarithmic term in the radiative correction due to photon is canceled by that due to family gauge bosons. In the Sumino model, it is essential that the left- and right-handed charged leptons e_{Li} and e_{Ri} are assigned to $\mathbf{3}$ and $\mathbf{3}^*$ of $U(3)$ family symmetry, respectively. (A similar fermion assignment has been proposed by Applequist, Bai and Piai [12].) Also, in his model, it is essential that the family gauge boson masses $m(A_i^j)$ are given by $m^2(A_i^j) \propto \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle \propto m_{ei} + m_{ej}$. As a result, we can obtain $K(\mu) = K^{pole}$. [However, it does not mean $m_{ei}(\mu) = m_{ei}^{pole}$. The cancellation takes place only for the term with $\log m_{ei}$ in Eq.(3.10).]

Stimulated by the Sumino model, in this paper, we assume $U(3) \times O(3)$ family symmetries [13]. The VEV relations (3.1) and (3.2) are derived from the following superpotential terms:

$$W_e = \mu_e (Y_e^{ij} \Theta_{ji}^e) + \lambda_e (\Phi_e^{i\alpha} \Phi_e^{T\alpha j} \Theta_{ji}^e), \quad (3.11)$$

$$W_u = \lambda_u [Y_u^{ik} E_{kj}^u (\Theta_u)_i^j] + \lambda'_u [(\Phi_u)_k^i (\Phi_u)_j^k (\Theta_u)_i^j], \quad (3.12)$$

where E^u takes a VEV form $\langle E^u \rangle = v_E \text{diag}(1, 1, 1)$. Here and hereafter, we denote fields whose VEV values are zeros as Θ_A ($A = e, u, \dots$). (Therefore, we can obtain meaningful VEV relations from SUSY vacuum conditions $\partial W / \partial \Theta_A = 0$, while we cannot obtain any relations from other conditions (e.g. $\partial W / \partial Y_f = 0$) because the relations always include $\langle \Theta_A \rangle$. For the time being, we assume that the supersymmetry breaking is induced by a gauge mediation mechanism (not including family gauge symmetries), so that our VEV relations among yukawaons are still valid even after the SUSY was broken in the quark and lepton sectors. In Eqs.(3.11) and (3.12), according to Sumino, we assume [8] that the gauge symmetry $O(3)$ is already broken at $\mu = \Lambda_{GUT}$.

Note that, for the VEV relations (3.3) – (3.5), we cannot use the similar prescriptions which were used for Eqs.(3.11) and (3.12), because we want a model without a cutoff Λ . We assume the following superpotential forms without Λ for the VEV relations (3.3) and (3.4):

$$W'_u = \lambda''_u (\Phi_u)_k^i P_u^{kj} \Theta_{ji}^u + \lambda'''_u \left[\Omega_E^{i\alpha} \Omega_E^{T\alpha j} + a_u \Omega_X^{i\alpha} \Omega_X^{T\alpha j} \right] \Theta_{ji}^u, \quad (3.13)$$

$$W_d = \mu_d Y_d^{ij} \Theta_{ji}^d + \lambda_d \left[\Omega_E^{i\alpha} \Omega_E^{T\alpha j} + a_d \Omega_X^{i\alpha} \Omega_X^{T\alpha j} \right] \Theta_{ji}^d, \quad (3.14)$$

together with

$$W_{E,X} = \mu_E \Omega_E^{i\alpha} \Theta_{\alpha i}^E + \lambda_E \Phi_e^{i\beta} E_{\beta}^\alpha \Theta_{\alpha i}^E + \mu_X \Omega_X^{i\alpha} \Theta_{\alpha i}^X + \lambda_X \Phi_e^{i\beta} X_{\beta}^\alpha \Theta_{\alpha i}^X. \quad (3.15)$$

where E , X and P_u have the VEV forms defined in Eq.(3.7). For such VEV forms of X and E , for example, we may consider the following superpotential forms [13]:

$$W_X = \lambda_X \det X \equiv \lambda_X \left(\frac{1}{3} \text{Tr}[X X X] - \frac{1}{2} \text{Tr}[X X] \text{Tr}[X] + \frac{1}{6} (\text{Tr}[X])^3 \right), \quad (3.16)$$

$$W_E = \lambda_E \text{Tr}[E E] \text{Tr}[E] + \lambda'_E (\text{Tr}[E])^3. \quad (3.17)$$

ξ_ν	$\tan^2 \theta_{\text{solar}}$	$\sin^2 2\theta_{\text{atm}}$	$ U_{13} ^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

Table 1: ξ_ν dependence of the neutrino parameters. The value of a_u is taken as $a_u = -1.78$ which can give reasonable up-quark mass ratios.

(Eq.(3.17) can uniquely lead to $\langle E \rangle \propto \mathbf{1}$, while Eq.(3.16) can only show that $\langle X \rangle$ is a rank 1 matrix. It is still an ad hoc assumption as well as in the previous models [3, 13] that the rank 1 matrix takes the democratic form given by Eq.(3.7) in the diagonal basis of $\langle \Phi_e \rangle$.)

For the VEV relation (3.5), we assume the superpotential

$$W_R = \mu_R Y_R^{ij} \Theta_{ji}^R + \lambda'_R [(\Phi_u)_k^i Y_e^{kj} + Y_e^{ik} (\Phi_u)_k^j + \xi_\nu (\Phi_u)_k^k Y_e^{ij}] \Theta_{ji}^R. \quad (3.18)$$

Here, the final term is somewhat different from the previous form (3.5). In the O(3) model, a term $(\Phi_u Y_e P_u + P_u Y_e \Phi_u)$ is allowed in addition to $(\Phi_u P_u Y_e + Y_e P_u \Phi_u)$, because these fields are $(\mathbf{5} + \mathbf{1})$ of O(3). However, in the present model, such a term is forbidden. Instead, we have added $\text{Tr}[\Phi_u] Y_e^{ij}$ to the term $(\Phi_u Y_e + Y_e \Phi_u)^{ij}$ as a new ξ_ν term in Eq.(3.18). This term is required [13] in order to fit the observed value of $\tan^2 \theta_{\text{solar}}$.

Thus, we can obtain the neutrino mass matrix Eq.(2.15) with

$$\langle Y_R \rangle = -\frac{\lambda_R}{\mu_R} (\langle \Phi_u \rangle \langle Y_e \rangle + \langle Y_e \rangle \langle \Phi_u \rangle + \xi_\nu \text{Tr}[\langle \Phi_u \rangle] \langle Y_e \rangle), \quad (3.19)$$

$$\langle \Phi_u \rangle \langle P_u \rangle = -\frac{\lambda_u'''}{\lambda_u''} (\langle \Omega_E \rangle \langle \Omega_E^T \rangle + a_u \langle \Omega_X \rangle \langle \Omega_X^T \rangle) = -\frac{\lambda_u''' \lambda_X^2}{\lambda_u'' \mu_X^2} v_X \langle \Phi_e \rangle (\langle E \rangle + a_u \langle X \rangle) \langle \Phi_e \rangle. \quad (3.20)$$

where, for simplicity, we have put $\lambda_E/\mu_E = \lambda_X/\mu_X$ and $v_E = v_X$. Numerical results for neutrino mixing parameters and up-quark mass ratios are identical with those given in Ref.[13]. We quote the numerical results from Ref.[13] in Table 1. Here, we have taken a value $a_u = -1.78$ which can give reasonable up-quark mass ratios:

$$\sqrt{\frac{m_u}{m_c}} = 0.04389, \quad \sqrt{\frac{m_c}{m_t}} = 0.05564. \quad (3.21)$$

The predicted values (3.21) are in good agreement with the observed values at $\mu = m_Z$ [14] $\sqrt{m_u/m_c} = 0.045_{-0.010}^{+0.013}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$.

In Eqs.(3.13) and (3.14), we have considered somewhat unbalanced assignments between Φ_e and Φ_u , i.e. $\Phi_e^{i\alpha} = (3, 3)$ and $(\Phi_u)_j^i = (8 + 1, 1)$ of $U(3) \times O(3)$. We may consider an alternative model with the same assignments $(\Phi_e)_j^i$ and $(\Phi_u)_j^i$. However, since we want to inherit the Sumino

	$\bar{\mathbf{5}}_i$	$\mathbf{10}_i$	$\mathbf{1}_i$	$\bar{\mathbf{5}}'_i$	$\mathbf{5}'^i$	$\bar{\mathbf{5}}''^i$	$\mathbf{10}''^i$	$\mathbf{5}''_i$	$\overline{\mathbf{10}}''_i$	$\bar{\mathbf{5}}_H$	$\mathbf{5}_H$
SU(5)	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{5}^*$	$\mathbf{5}$	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{10}^*$	$\mathbf{5}^*$	$\mathbf{5}$
SU(3)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
O(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Σ_3	Σ_2	Y_e	Y_d	Y_u	Y_R	Φ_e	Φ_u	Ω_E	Ω_X
$\mathbf{24} + \mathbf{1}$	$\mathbf{24} + \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{3}^*$	$\mathbf{8} + \mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}^*$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$

E^u	P_u	E	X	Θ^e	Θ_u	$\Theta^{u'}$	Θ^d	Θ^E	Θ^X	Θ^R
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{8} + \mathbf{1}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{6}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$

Table 2: Fields in the present model and their SU(5)×U(3)×O(3) assignments.

mechanism [8] for the charged lepton mass relation, we adopt the assignment $\Phi_e^{i\alpha}$. On the other hand, we do not adopt the assignment $\Phi_u^{i\alpha}$, because if we adopt such the assignment, we are forced to modify the structure of W'_u given in Eq.(3.13) and W_R given in Eq.(3.18) into more complicated forms in order to express these terms without Λ .

In Table 2, we list assignments of SU(5)×U(3)×O(3) for all fields in the present model. The model is anomaly free in SU(5), while it is not so in the U(3) gauge symmetry. Since the anomaly coefficients of $\mathbf{3}$, $\mathbf{6}$ and $\mathbf{8}$ of SU(3) are $A(\mathbf{3}) = 1$, $A(\mathbf{6}) = 7$ and $A(\mathbf{8}) = 0$, the sum of the anomaly coefficients A is $\sum A = (42 - 29)A(\mathbf{3}) + (5 - 5)A(\mathbf{6}) = 13$. In order that the model is anomaly free for U(3) family symmetry, we need further fields which give $\sum A = -13$, so that we may consider, for example, A^{ij} , $B^{i\alpha}$ and $C^{i\alpha}$. However, for the time being, we do not specify the roles of those fields A , B and C in the model.

R charges of these fields in the yukawaon sector must satisfy the following relations:

$$R(Y_e) = 2R(\Phi_e) \equiv r_e, \quad (3.22)$$

$$R(Y_u) = 2R(\Phi_u) - R(E^u) \equiv r_u, \quad (3.23)$$

$$R(Y_d) = R(\Phi_u) + R(P_u) = 2R(\Omega_X) = 2R(\Omega_E) = 2\left(r_e + \frac{2}{3}\right), \quad (3.24)$$

$$R(Y_R) = R(\Phi_u) + R(Y_e) \equiv r_R. \quad (3.25)$$

In these assignments, since we have still free parameters, we do not give numerical assignments of R charges for these fields.

Finally, we would like to comment on R parity assignments. Since we inherit R parity assignments in the standard SUSY model, R parities of yukawaons Y_f (and also Θ_f , $\Phi_{e,u}$, E , \dots) are the same as those of Higgs particles (i.e. $P_R(\text{fermion}) = -1$ and $P_R(\text{scalar}) = +1$),

while $(\bar{\mathbf{5}}'' + \mathbf{5}'')$, $(\mathbf{10}'' + \overline{\mathbf{10}}'')$, \dots are assigned to quark and lepton type, i.e. $P_R(\text{fermion}) = +1$ and $P_R(\text{scalar}) = -1$.

4. Energy scales

Masses of the additional particles $f' = (\bar{\mathbf{5}}' + \mathbf{5}')$ and $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'') + (\mathbf{5}'' + \overline{\mathbf{10}}'')$ are given by

$$m(f') \sim \langle \Sigma_{2,3} \rangle, \quad m(f'') \sim M_{5,10}. \quad (4.1)$$

The existence of these additional particles does not affect the value of Λ_{GUT} in the minimal supersymmetric standard model (MSSM), so that the scale of Λ_{GUT} is still given by

$$\Lambda_{GUT} = 2 \times 10^{16} \text{ GeV}. \quad (4.2)$$

Since the VEV forms of $\Sigma_{2,3}$ break $SU(5)$, it seems to be natural to consider

$$\langle \Sigma_2 \rangle \sim \langle \Sigma_3 \rangle \sim \Lambda_{GUT} \quad (4.3)$$

On the other hand, if we consider a lower value of $M_{5,10}$, the gauge coupling constants $\alpha_3(\mu)$ will blow up before μ reaches the GUT scale Λ_{GUT} . We have a constraint

$$M_{5,10} \geq 10^{12} \text{ GeV}. \quad (4.4)$$

Masses of the quarks and leptons are given as

$$M_u = y_u y_{10} \frac{\langle Y_u \rangle}{\bar{M}_{10}} \langle H_u^0 \rangle, \quad M_{e,d} = y_{e,d} y_5 \frac{\langle Y_{e,d} \rangle}{\bar{M}_5} \langle H_d^0 \rangle, \quad (4.5)$$

from Eqs.(2.7) and (2.8), so that we have constraints

$$\langle Y_u \rangle / \bar{M}_{10} \sim 1, \quad \langle Y_e \rangle / \bar{M}_5 \sim \langle Y_d \rangle / \bar{M}_5 \sim 10^{-1}, \quad (4.6)$$

which means

$$(\bar{M}_{10})^2 \sim (M_{10})^2 \sim \langle Y_u \rangle^2, \quad (\bar{M}_5)^2 \sim (M_5)^2 \gg \langle Y_{e,d} \rangle^2, \quad (4.7)$$

respectively.

Taking the constraints (2.17) and (4.4) into consideration, we assume that particles f'' have masses of the order of

$$M_5 \sim M_{10} \sim 10^{14} \text{ GeV}, \quad (4.8)$$

so that an energy scale at which $U(3)$ is broken is

$$\langle Y_f \rangle \sim \langle Y_R \rangle \sim \Lambda_{fam} \sim 10^{13} \text{ GeV}. \quad (4.9)$$

For reference, we illustrate the behaviors of the gauge coupling constants $\alpha_i(\mu)$ ($i = 1, 2, 3$) for the case $M_{5,10} = 10^{14} \text{ GeV}$ in Fig.1.

As seen in Table 2, we have many $U(3)$ non-singlet fields in the present model, so that the model does not give an asymptotic free theory. The gauge coupling constants of $U(3)$ will blow up before μ reaches $\mu = \Lambda_{GUT}$ if we take a lower value of Λ_{fam} . In this paper, we adopt a

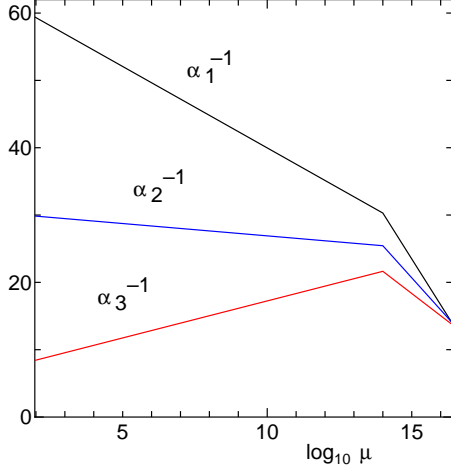


Figure 1: Behaviors of gauge coupling constants α_i^{-1} ($i = 1, 2, 3$) in the case of $M_{5,10} = 10^{14}$ GeV. For simplicity, we have neglected the SUSY breaking effects at $\mu \sim 10^3$ GeV in this figure.

scenario based on Eq.(4.9). Then, since the scale $M_{5,10}$ is very high, the family gauge coupling constants α_{fam} does not blow up from $\mu = M_{5,10}$ up to $\mu = \Lambda_{GUT}$.

We do not discuss the behaviors of gauge coupling constants above $\mu = \Lambda_{GUT}$ because we have no scenario at $\mu > \Lambda_{GUT}$ at present.

5. Concluding Remarks

In conclusion, we have investigated a yukawaon model which is compatible with an SU(5) GUT scenario. Since yukawaons are SU(5) singlets, the existence of the yukawaons do not affect the SU(5) GUT model, so that we can inherit the successful results in the SU(5) GUT and we also inherit the current problems in the minimum SU(5) GUT scenario. We optimistically consider that those problems peculiar to the SU(5) scenario will be resolved in a model based on a higher GUT symmetry and/or on extra-dimension scenario.

In the present model, we have the following matter fields:

$$(\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1})_i + (\bar{\mathbf{5}}'_i + \mathbf{5}'^i) + (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i. \quad (5.1)$$

We do not consider $\mathbf{1}''_i + \mathbf{1}''^i$. The particles $f' = (\bar{\mathbf{5}}'_i + \mathbf{5}'^i)$ and $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$ have masses of the orders of $\Lambda_{GUT} \sim 10^{16}$ GeV and $M_{5,10} \sim 10^{14}$ GeV, respectively. The U(3) family symmetry is broken at $\mu = \Lambda_{fam} \sim 10^{13}$ GeV, whose value has been settled by a neutrino seesaw mass and no blowing up condition of the conventional gauge coupling constants.

The most notable result in the present SU(5) compatible model is that the model naturally lead to a model without Y_ν in which Y_e plays a role of a substitute for Y_ν , although it was an ac hoc assumption in the previous yukawaon model [3].

It is also worthwhile to notice that the present model is a model without a cutoff scale Λ differently from the past yukawaon models, and the factors \bar{M}_{10} and \bar{M}_5 in the seesaw expressions (2.7) and (2.8) have family-dependence. In the past yukawaon models, since the model have been based on an effective theory with a cutoff parameter Λ , we are obliged to consider $\langle Y_u \rangle / \Lambda \sim 1$ in order to explain the observed fact $m_t \sim \langle H_u^0 \rangle$. However, in the effective theory with the scale

Λ , it was unnatural to consider $\langle Y_u \rangle \sim \Lambda$. In the present model, since the cutoff parameter Λ is replaced with $\bar{M}_{10} = M_{10}(1 + \langle Y_u \rangle^2/M_{10}^2)$, we can safely choose $\langle Y_u \rangle \sim M_{10}$. Besides, we can expect visible effects in the phenomenology of the up-quark mass matrix due to $\bar{M}_{10} \neq M_{10}$, although the effects due to $\bar{M}_5 \neq M_5$ in the charged lepton and down-quark sectors are negligibly small. Phenomenological investigation based on the present model with $\bar{M}_{10} \neq M_{10}$ will be given elsewhere.

Regrettably, we have failed to build a model in which U(3) family gauge boson effects are visible (for instance, Ref.[15]). It seems to be hard to embed a lower scale of Λ_{fam} (e.g. $\Lambda_{fam} \sim 10^{7-8}$ GeV [13]) into the present SU(5) compatible yukawaon model. This is still our future task.

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